Paper

Utilization in Artificial Ego of Quantum Calculations Based on Anti-Einstein Field Hypothesis

反アインシュタイン場仮説にもとづく量子計算の人工自我への活用

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In this paper, the extended Riemann sphere of the anti-Einstein field hypothesis is used to clarify the mechanism by which quantum gates emerge. In addition, this paper shows that artificial nerves exhibiting emergent phenomena are achievable as an engineering application of said mechanism.

In general, the Bloch spheres and Riemann spheres used for quantum gate spinor representation cannot handle attribute inversion at poles. However, our proposed extended Riemann sphere is capable of handling attribute inversion at the poles where Riemann spheres and anti-Riemann spheres are connected. Thus, it is anticipated that emergence will be controllable using quantum gates based on the extended Riemann sphere.

With this in mind, we designed an emergence-imitating artificial nerve model in an extended Riemann sphere, and implemented and tested the model on a computer. In addition to confirming emergence phenomena in the extended Riemann sphere, we discovered that the behavior of transfer functions, which diverge and become unmanageable in control engineering, gave rise to various sigmoid functions.

Therefore, we use the extended Riemann sphere as a basis to clarify why quantum gates emerge, and, in consideration of the results obtained by using the extended Riemann sphere to confirm the actions of the emergence-imitating artificial nerve model, mathematically express the implications for black holes, white holes, emergence, artificial nerves, and artificial ego.

Keywords: Bloch sphere, Riemann sphere, quantum gate, division by zero, emergence, artificial nerve, artificial ego, quantum computing

1. Introduction

As of June 2021, production of the humanoid communication robot “Pepper” has been halted for temporary adjustments. First released in 2015, Pepper has remained in production for a long time for a humanoid robot, yet it is not well known that Pepper now comes equipped with an early-stage artificial ego[1] which, in addition to recognizing human emotions, draws on other sensor information to generate emotions such as “like” and “dislike” in the robot itself. This was the world’s first attempt at installing an artificial ego in a commercial device.[2]

A major problem in this artificial ego project was whether the robot would exhibit “free emergence” like a human. Emergence is one of the most important functions of artificial ego. However, at the time, the mathematical models for emergence were not clear and there were no mathematical models of morality to control the selfish behavior resulting from emergence. It is for these two reasons that emergence was not then achievable.

In response to this problem, we postulated, as in Figure 4, that “there is a world we cannot currently perceive, and something will emerge due to its influence”, and published, in DHU JOURNAL in 2019, an extended Riemann sphere that connects at the poles where a Riemann sphere (positive world) and an anti-Riemann sphere (anti-world) undergo attribute inversion.[3] In the extended Riemann sphere, for example, the north pole of a Riemann sphere surface (infinity) defining stereographic projection on a complex plane to which an ideal point has been added (expanded complex plane) is connected, as in a Klein bottle or Möbius strip, to the south pole of a converse Riemann sphere surface (anti-infinity, anti-north pole) that is related to the former sphere by attribute inversion. Obverse and reverse surfaces related by attribute inversion are connected to each other in any Klein bottle or Möbius strip, but in the proposed extended Riemann sphere, the connection is between an infinity and anti-infinity that are related by attribute inversion. In this extended Riemann sphere, “emergence” is the phenomenon whereby an infinity formed in a null set of the positive world, and an anti-infinity formed in an anti-null set of the anti-world, overlap and produce 1.[3]

We also illustrated basic designs for the artificial ego, and presented various calculations required by emergence. More precisely, as basic designs for the artificial ego, we provided examples of how homeostatic resilience in the positive world and anti-world can be expressed in functions, in association with an Emotion Map.[4] In addition, as calculations required by emergence, we published, in DHU JOURNAL in 2020, a cut operation for explicitly and flexibly performing a partition of unity, a dynamization operation (오) for linking mathematical phenomena eventuated by the cut operation, a superposition operation (오^f) for expressing dynamization operations in which emergence occurs, and converse (*rev*) and inverse (*inv*) operations for explicitly differentiating and calculating converse and inverse with respect to an anti-world produced in a partition of unity.[4] Moreover, we have already applied, through the University of Tokyo, for a patent for a quantum gate based on the extended Riemann sphere.

This paper clarifies the operating principles of emergent quantum gates based on the extended Riemann sphere. In addition, it applies these operating principles to implement emergence-imitating artificial nerves and show that artificial ego emergence occurs at the artificial nerve level.

Table 1: Novel Calculation Standards

Novel Calculation	Operator	Definition of Calculation	Result of Calculation	Special Notes
Cut operation	cut	Physical quantity E corresponding to sum 1 is represented using base vector or base function N^n as $a_nN^n+a_{n-1}N^{n-1}+a_1N^1+a_0N^0=\mathrm{E}\leftrightarrow 1$. If N is constituted by a continuous quantity, the cut operation is defined as $1\ cut\ n=a_nN^n+a_{n-1}N^{n-1}+a_1N^1+a_0N^0$. However, in a dynamization operation, this can also be expressed as $1\ cut\ n=$ $a_nN^n\text{오}a_{n-1}N^{n-1}\text{오}\ldots\text{오}a_1N^1\text{오}a_0N^0$. If a_nN^n is constituted by a discrete quantity, the cut operation is defined as $1\ cut\ n=a_nN^n+a_{n-1}N^{n-1}+\ldots+a_1N^1+a_0N^0$. On the right side of the equation, $a_nN^n\text{오}a_{n-1}N^{n-1}\text{오}\ldots\text{오}a_1N^1\text{오}a_0N^0$ or $a_nN^n+a_{n-1}N^{n-1}+\ldots+a_1N^1+a_0N^0$ is referred to as "*shiki*". However, the range within which $n\in\mathbb{N}$ can be obtained is $-\infty\leq n\leq\infty$. Moreover, when $n=\pm\infty$ (infinite dimension), the physical quantity E is represented as $\int_0^\infty a(x)N(x)dx=E\leftrightarrow 1$.	*shiki* *識*	Encompasses ÷ (partitive)
Dynami-zation operation	오	Dynamization operation 오 is defined as a calculation to obtain the solution $a_nN^n+a_{n-1}N^{n-1}+\ldots+a_1N^1+a_0N^0=\mathrm{E}$. Here, the set of solutions satisfying *shiki* is called "*tai*".	*tai* *態*	Straight lines and flat planes are also *tai*.
Superpo-sition operation	오^f	$(\emptyset:\mathrm{anti}\cdot\emptyset)*(\infty+\mathrm{anti}\cdot\infty)=(0\equiv\infty)=1$ However, $\sum_{i\in Rieamann}a_iN^i\rightarrow\infty$ and $\sum_{j\in anti\cdot Riemann}a_jN^j\rightarrow anti\cdot\infty$	*ketsu* *結*	Encompasses ÷0
Inversion operation	Dualism: *not*. Trialism: *inv/rev*.	In dualism, $\sum_{i\in foreside}a_iN^i\perp\sum_{j\in backside}a_jN^j$ is $not\cdot\left(\sum_{i\in foreside}a_iN^i\right)=(a_nN^n+\ldots+a_0N^0)-\sum_{i\in foreside}a_iN^i=\sum_{j\in backside}a_jN^j$ In trialism, $\sum_{i\in foreside}a_iN^i\perp\sum_{j\in rev}a_jN^j\perp\sum_{k\in inv}a_kN^k$ is $rev\cdot(a_nN^n+\ldots+a_0N^0)=\sum_{j\in rev}a_jN^j$ $=(a_nN^n+a_{n-1}N^{n-1}+\ldots+a_0N^0)-\sum_{i\in foreside}a_iN^i-\sum_{j\in inv}a_jN^j$	*inv裏* *rev逆*	Trialism also encompasses *not*. Definition of *not* is the same as for dualism.

The novelty of this paper rests on the following two points.

The first point of novelty is our mathematical clarification of the fact that emergence is caused by the infinity produced by division by zero. In information engineering, dividing by zero on a computer produces an error. In mathematics, dividing by zero in a zero ring returns 0=1 and is trivial. Alternatively, with division by zero in a Riemann sphere, an ideal point on an expanded complex plane is translated by stereographic projection to infinity on the Riemann sphere, resulting in a tautology. In physics, the approach is that the infinity resulting from division by zero does not exist. In the inverse square of distance law and Schwarzschild solutions, despite strong suggestion that division by zero is an entity that cannot be ignored, neither Einstein, Penrose, Hawking, nor any other eminent physicist has ever ventured a clear interpretation of division by zero. This paper shows, through concise mathematics, that division by zero is the trigger of the emergence phenomenon.

The second point of novelty is that these newly defined calculations make clear that the emergence phenomenon can be handled in a unified manner. Clarifying the definitions of extended Riemann sphere, inversion operation, and superposition operation makes it possible to recreate and even control the emergence phenomenon on existing computers. That is to say, using the inversion operation to preserve the essence of a function in infinite dimensional vector space makes it possible to use the superposition operation to distort and condense to a single point the space over which the ideal point stretches and develop the essence of the function using anti-space coordinates. Furthermore, this approach shows that even if the function disappears at infinity, a function is simultaneously generated from infinity. In other words, this clearly shows a state in which positive and anti-infinite dimensional vector spaces preserve essence of function, and the temporal axes of the functions that are generated and disappear in the two spaces are superposed. To give an analogy, a memory preserved in space would be an Akashic record, and the novel calculation would indicate a five-dimensional existence.

2. Assumptions

2.1 Updates to Novel Calculation Standards

Using the extended Riemann sphere, wave and particle mechanisms can be understood as in Figure 1.

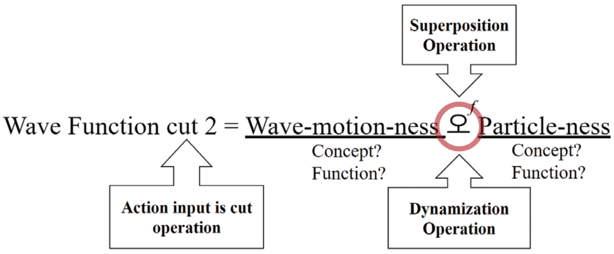

Figure 1: Relationship of cut operation, superposition operation, and dynamization operation in wave function

Various kinds of information can be obtained from wave functions depending on the measurement and interpretation, and wave-motion-ness and particle-ness are examples of such information. It is the action of measurement or interpretation that defines the frame for obtaining outputs that are limited to wave-motion-ness and particle-ness from among any number of nature. The frame is defined by the input to the function resulting from these actions (action input), and by performing a cut operation of a wave function in such frame, it is possible to obtain a superposition of wave-motion-ness and particle-ness. In mathematics, the cut operation can be expressed as a partition of unity which is defined in the topological space theory. In physics, because the sum E of physical quantity corresponding to a unity need not be 1, the cut operation can be expressed as a partition of E by using base vectors or base functions. Consequently, the superposition of wave-motion-ness and particle-ness resulting from the cut operation acts complementarily while preserving the sum. The polynomial expression for maintaining the sum obtained in the cut operation is referred to as *"shiki" (識)*, and the set of solutions satisfying *shiki* is referred to as *"tai" (態)*. In this context, we will define positive and anti in order to describe the details of *shiki*. "Positive" (orthodox) is defined as the property of the space spanned by each vector of *shiki* that relate to observation extend, on the basis of the projection hypothesis. Likewise, "anti" is defined as the property of the space spanned by each vector of *shiki* that do not relate to observation extend. Furthermore, "superposition operation" is defined as the calculation for superposing positive and anti in infinite dimensions. The sum produced by the superposition operation is referred to as *"ketsu"(結)*. Finally, the "inversion operation" is defined as the operation for calculating the terms of the anti. The cut operation, dynamization operation, superposition operation, and inversion operation are detailed in Table 1. It should be noted that results of addition, subtraction, multiplication, and division operations are referred to as "sum", "difference", "product", and "quotient" respectively; likewise, the results of the cut, dynamization and superposition operations are referred to as *"shiki"*, *"tai"* and *"ketsu"* respectively.

In dualistic logic, only "not" is anti, but in pluralistic logic, inside (*rev*, reverse) and back (*inv*, inverse) are anti in addition to *not*. The reason "inside" is conceived as reverse is, to give an admittedly limited example, that in the sentence "you can't reverse the clothes because they're not reversible", "reverse" means turning the clothes inside out. In Japanese, turning clothes inside out typically means flipping the inside and outside, and does not mean "reverse" in the sense of forward and backward. On the other hand, the reason inverse is conceived in terms of "back" is that the not P→not Q concept of converse of contrapositive (inverse) and the concept of back (backside) are closely related linguistically in Japanese. If we think of bits as coins, the 1 on the front/obverse surface (foreside) is flipped over and becomes a 0 on the back/reverse surface (backside). In other words, the concept of converse of contrapositive (inverse) is connected to the concept of back surface (backside) via this concept of flipping (inversion). Thus, as shown in Figure 2, when we think in terms of a planar two-dimensional cut operation, front/obverse and back/reverse can be obtained. Likewise, when we think in terms of a volumetric three-dimensional cut operation, front/obverse, inside, and back/reverse are obtained.

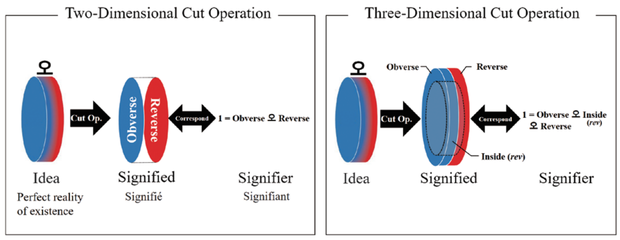

Figure 2: Difference between two-dimensional cut operation and three-dimensional cut operation

Just as "inverse" means different things in logic, algebra, and geometry, caution is needed with the extended Riemann sphere and novel calculation standards to ensure there is no confusion regarding *inv*. In the extended Riemann sphere, the *con* (converse) of the Riemann sphere is the converse Riemann sphere, and the *inv* (inverse) of the Riemann sphere is the inverse Riemann sphere. Meanwhile, in the novel calculation standards, the *inv* (inverse) of the Riemann sphere is the converse Riemann sphere, and the *rev* of the Riemann sphere is the inverse Riemann sphere.

There are three major differences between division and cut operations. The first is that division is limited to partitive division (equal division), whereas cut operations are not so limited. The second difference is that partitioning of concepts cannot be represented in division, but can be represented in cut operations; when partitioning of concepts is represented in a cut operation, the terms of the cut operation $a_0N^0,a_1N^1,\ldots,a_{n-1}N^{n-1},a_nN^n$ are distributed representations of words. The third difference is that the functions (roles) exercised by numbers, volumes, concepts, subjects, acts, and results are difficult to explain using division, but can be explained according to definitions in cut operations. For example, if a cut operation is viewed in terms of such functions, the subject (input) of the function may be not only a number represented by a discrete quantity, a volume represented by a continuous quantity but a concept. In addition, the function of the cut operation can be explained as the partitioning of subjects; this partitioning function can be considered a partitioning action; and the result (output) of the function will be an equation (*shiki*) representing dynamic equilibrium that partially changes while preserving a sum is output.

2.2 Computers and Consciousness

For the extended Riemann sphere to give computers consciousness, the calculation processes of those computers must expand from a dualistic logic model to a pluralistic model. This is because, when looking from the perspective of wave or particle, just as nature does not make a dualistic choice between wave and particle, but a pluralistic choice in which wave-motion-ness and particle-ness are superposed, consciousness, which is a natural phenomenon, similarly does not make a dualistic choice. For example, when humans and other advanced primates believe that a thesis A→B holds, they have a biased tendency to believe that its converse B→A also holds.[5] In other words, it is possible for superposition of thesis and converse to occur in consciousness. However, in logic based on dualism, only propositions whose truth values match, such as contrapositives, are treated as equivalent. It is not clear what causes us to conceive of the superposition of propositions. Thus, because thesis and converse have inconsistent truth values, they are simply treated as different things. There are some humans who tend to distinguish thesis and converse in their thinking, but this behavior is similar to the behavior model of primitive animals[5][6] and insects.[7] Therefore, the above-described tendency toward biased thinking does not exist in insects and computers, and can be considered a bias that is observed in humans and other advanced primates precisely because they have the sophisticated function of consciousness. Further, although, for example, memory and consciousness reciprocally form one another, in logic based on dualism, the superposition of memory and consciousness cannot be represented. More specifically, if the proposition variable X is "formation of consciousness state A" and the proposition variable Y is "formation of memory B", the thesis X→Y can be defined as "if consciousness state A is formed, then memory B is formed". Conversely, the converse Y→X is the proposition "if memory B is formed, then consciousness state A is formed". Generally speaking, the truth claims of a thesis and its converse do not match, and dualism thus regards these two propositions as distinct. Such an approach lacks the depth that would allow it to use *rev* to treat propositions as the same while also differentiating them.

In response to this problem, we have constructed the new logical model necessary to give computers deep and sophisticated consciousness. The foundation of our new logical model is not the dualism rooted in the Old Testament that regards existences as entirely separate, but a pluralism rooted in Japanese culture that regards existences as superposed.

2.3 Construction of a Pluralism-Based Logic

The foundation of von Neumann computers is dualism. The foundation of current quantum computing is a dualism in which an original multiaxial superposition is turned into a superposition of 0 and 1. This happens because the foundation of logic is dualism.

Our goal, however, is the development of an emergent artificial ego. Codes of value for behavior are necessary when a machine will choose its own behavior via emergence.

Thus, as the foundation of such a logic, we turned our attention propositional logic and considered using propositional logic to express a code of values for behavior. In propositional logic, if x is a behavior and P and Q are sets of behaviors, a code of value for the behavior can be expressed as "(x∈P) →(x∈Q)". Below, the proposition "(x∈P) →(x∈Q)" is represented simply as (P→Q). Likewise, "code of value for behavior" is shortened to "behavior code".

As a concrete example, let us consider the behavior code of the samurai. If the set P is "acts of justice" and the set Q is "means of living", then the samurai behavior code can be expressed by the thesis (P→Q). According to the samurai behavior code, any of the acts of justice exemplified by fidelity to one's lord will be a "means of life". In other words, justice is the means by which life is fulfilled.

The converse of the samurai behavior code can be expressed as (Q→P). This converse holds that anything that is a "means of living" is "just". This converse can be thought of as expressing an "egoistic set (β)" behavior code of people who do not choose the means by which they live. In this context, people who always adopt the converse behavior code would be called demons [*oni*]. The converse is a demonic behavior code. The demons always seem to expand the concept of justice to secure their means of survival.

The inverse obtained by inverting the samurai behavior code can be expressed as (not-P→not-Q). In other words, this is a behavior code where nothing that is not an act of justice can be a means of living. We can think of the inverse proposition as expressing a culture in which a samurai that has acted unjustly commits *seppuku* out of shame. In this case, the shame is an egoistic act.

The contrapositive of the samurai behavior code can be expressed as (not-Q→not-P). This is a behavior code in which nothing that is not a means of living can be a just act. According to such a behavior code, suicide is not a just act because it is not a means of living. We might also think of this code, which regards suicide as unjust, as expressing the widely known principle of monotheistic religions that "any human who commits suicide will go to hell". However, according to traditional Japanese codes of value, the *seppuku* of the samurai is not mere suicide, and needless to say, is not evil. In this we find a depth that cannot be expressed dualistically. Even monotheism allows for thinking that distinguishes egoistic suicide from altruistic suicide, so the story is not as simple as is commonly thought.

Thus, an "altruistic set (α)" can be conceived as an antipode of the "egoistic set (β)" of the converse, and we can consider this α code of value to exist within the samurai. When *rev* is used, α=rev-(P→Q) and β=rev-(Q→P). When *rev* is used to add depth to the contrapositive, (not-Q→not-P) attains a dual nature comprising α=rev-(not-Q→not-P) and β=rev-(not-Q→not-P). Thus, the suicide that is regarded as evil in typical monotheistic religions can be interpreted as evil in the sense of being the execution of "egoistic set (β)". Likewise, samurai *seppuku* can be interpreted as good in the sense of being the execution of "altruistic set (α)". In other words, in a single contrapositive, there are different essences (*rev* of the proposition), and we understand this difference of essence to be determined according to lineage – in other words, the state before inverse and converse were calculated.

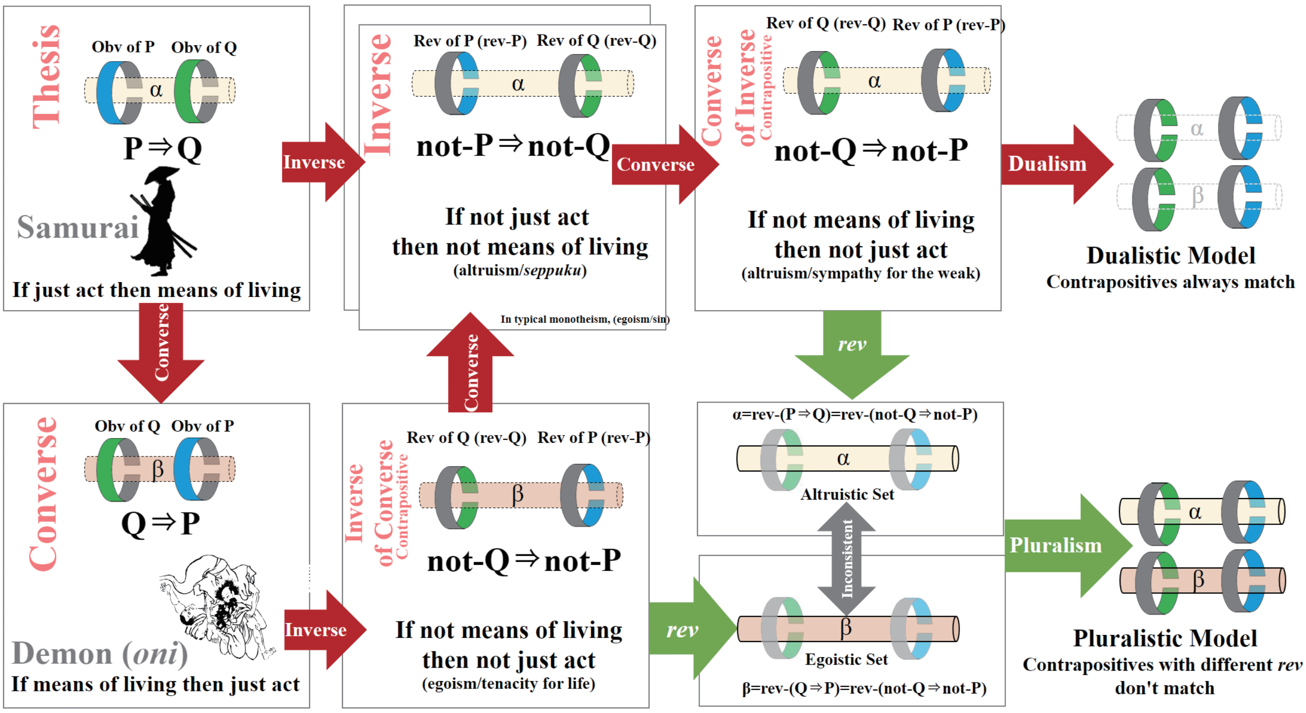

Figure 3: Using propositional logic to explain the difference between dualism and pluralism

We also believe that this relationship between *rev* and *not* provides a hint to the uniquely human thought principle which assumes the converse to be true. In dualism, a thesis and its converse are different entities, but in a pluralism that accounts for *rev*, the parameters implicit in the propositions can match. Thus, we believe there is a principle of thought where, for example, when a thesis and the *rev* of its converse match, a person will at once mix up and seek to unify these. The existence of such a principle can also be predicted from the fact that most invention and emergence is born of human preconceptions, and comes into the world by accident.

In light of the foregoing, the difference between dualism, which represents behavior codes using only *not*, and pluralism, which represents behavior codes using both *not* and *rev*, is clear, as shown in Figure 3. The distinction can be drawn according to the essential difference whereby, for example, suicide falling under the inverse is always evil under dualism, whereas under pluralism, egoistic suicide is evil but altruistic suicide is good. Similar claims may be made about the converse of the inverse (contrapositive). Considered pluralistically, the behavior code holding that something is not a just act if it is not a means of living can be interpreted, according to the "altruistic set (α)", as a standard that counsels mercy for the poor and the weak. Any justice that would deny the weak in their struggle for life can thus be denied on altruistic grounds. Meanwhile, according to the "egoistic set (β)", this behavior code can be interpreted only in terms of clinging to life. This set expresses a self-centered view according to which no behavior is just which is not a means of securing one's own life. Dualism cannot express this difference of fundamentally latent parameters. Therefore, unless a pluralistic model rooted in pluralism is constructed in a behavior code, morality and emotion will be unrepresentable, and as a result, behavior codes that can empathize only with some people will be all that can be implemented in machines.

2.4 Pluralistic Model Necessary to Achieve Morality and Emotion

In this section, we refer to the pure and fundamental motives preceding the reception of external influences as "emotion", and consider the representation of robot emotion using vectors. Vectors representing the fundamental motives of robots are referred to as "emotion vectors", which are defined as expressing, as one point in an n-dimensional vector space, the direction of the next state to be achieved in a robot's state space.

A robot behaving in accordance only with emotion vectors will be in danger of inclining towards self-centeredness and egoism. Mechanisms for controlling emotion vectors are therefore necessary; for example, mechanisms restricting the very generation of emotion vectors are one such possibility. Pepper, which was mentioned in the introduction to this article, is equipped with an emotion vector generation function and has emotions such as like and dislike for people and environments. However, functions that impulsively exhibit hatred for an owner are likely to be deemed industrially useless and commercially dangerous by manufacturers. By "industrially useless", we mean that a robot is of no use if it does not follow instructions, and by "commercially dangerous", we mean that commercial value will be destroyed if, for example, impulsive behavior by the artificial ego is spread online through malicious videos. As a result, systems have been adopted in operational apps in which emotion vectors are deactivated and then activated temporarily in only certain communication or game apps where impulsive expression is permitted; the reason that it was not well known that the early-stage artificial egos had been installed is that operations were restricted. Another option worthy of consideration is a mechanism that would use morality vectors to control emotion vectors – that is to say, a mechanism defined by "emotion vector + morality vector = desire vector", where the robot acts on the basis of the desire vector. This method would allow the robot to be controlled by the morality vector such that the emotion vector is oriented toward sympathizing emotionally with others.

Incidentally, as discussed above, when behavior codes are defined dualistically with consideration only for *not*, one encounters a problem in that, for example, altruism and egoism are not separable when those behaviors are superficially the same. Thus, it is necessary to conceive of behavior codes on the basis of a pluralism that also considers *rev*. Research into whether "only-*not*" behavioral codes or behavior codes that also consider *rev* lead to moral or emotional judgments, and to sympathy with others, is therefore also necessary.

3. Emergent Quantum Gates

We postulate, as in Figure 4, that "if there is a world we cannot currently perceive (anti-world), something will emerge due to its influence".

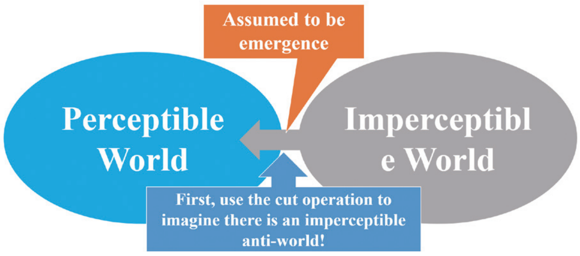

Figure 4: Existence of imperceptible anti-world

The mathematical model that explains the anti-world is an extended Riemann sphere that connects a Riemann sphere and anti-Riemann sphere at the poles where attribute inversion has occurred.[3] In the extended Riemann sphere, the anti-world is represented as the converse Riemann sphere and inverse Riemann sphere. In addition, "emergence" can be defined as the phenomenon whereby an infinity formed in a null set (KU) of the positive world, and an anti-infinity formed in an anti-null set (anti-KU) of the anti-world, overlap and produce 1.[4] By means of the extended Riemann sphere, black holes and white holes are sketched as entities whose obverse and reverse are stuck back-to-back to each other, and appear as mechanisms related to emergence.

In this section, we will make clear, on the basis of the mathematical model of our proposed extended Riemann sphere, that black holes and white holes are related to emergence mechanisms, and will propose a basic design for an emergent quantum gate.

3.1 Relationship between Black Holes, White Holes, and Emergence

In Einstein's general relativity theory,[8] the event horizon and singularity are known problems in relation to black holes.[9] Specifically, when a black hole dies due to evaporation from Hawking radiation, because Hawking radiation originating in a vacuum does not contain matter information, the black hole dies without it being known where the matter information within the event horizon went.

In a Riemann sphere, the infinity resulting from division by zero can be handled as one point on a sphere surface. However, no interpretation of infinity is ventured in a Riemann sphere, and because there is nothing after infinity, the collapsed matter information must, for example, return along with the Hawking radiation.[10] In other words, a Riemann sphere can be regarded as treating ∞ as the reciprocal of zero and division by zero as the same as multiplication by infinity, and $z \in C \backslash 0$, in which 0 is excluded from the complex plane, defines $z/0=\infty$ and $z \cdot \infty=\infty$. In addition, stereographic projection is defined such that points on a complex plane to which an ideal point has been added correspond one-to-one to points on the Riemann sphere; specifically, stereographic projection is defined such that the ideal point on the complex plane corresponds to the north pole of the Riemann sphere. However, no interpretation of the north pole – for example, of the height of the north pole – is ventured, and nothing is connected to the north pole. Thus, what comes after infinity must also be considered in Riemann spheres.

At the same time, Riemann spheres also present the problem of being incapable of conceiving a partition of infinity. In what follows, we will begin by explaining quotative and partitive division, and then show that in a Riemann sphere, a partition of infinity cannot be conceived in the manner of quotative division.

To begin, in arithmetic education, partitioning of a whole can be conceived in terms of quotative or partitive division. Quotative division is division to determine how many a whole becomes when divided by a certain number. For example, quotative division is used when calculating the answer to, "If 6 total candies are divided into groups of two, with how many people can the candy be shared?" Partitive division is used to determine the number per group when a whole is divided among a given number of groups. For example, partitive division is used when calculating the answer to, "If 6 total candies are divided among 3 people, how many candies will each person receive?" In both quotative and partitive division, the whole is a discrete quantity, but for the purposes of this paper, the whole may be a continuous quantity as well. However, in quotative division, the quotient must be a discrete quantity, and the custom is to treat fractions as remainders. For example, when 5.5 apples are divided into groups of 2.5 by quotative division, the apples can be shared with 2 people and there is a remainder of 0.5. The divisor in partitive division is generally a discrete quantity.

Next, we will show that, when a case where distance is zero in an equation containing terms dividing mass by distance (hereinafter, "zero distance"), as in the law of universal gravitation, is considered in terms of quotative division, a partition of infinity is inconceivable even using a Riemann sphere. Because a Riemann sphere defines ${}^{\infty}/_{0}=\infty$, conceiving ${}^{\infty}/_{0}$ from the perspective of quotative division would mean that infinity is divided by the quantity of zero, and thus can be divided into infinity boxes. Conversely, although infinity should be the result when zeroes are extracted as quantities from infinity boxes and added together, in a Riemann sphere, $0 \cdot \infty$ is undefined, and even if ${}^{\infty}/_{0}=\infty$ is established as in quotative division, $0 \cdot \infty=\infty$ is not established. Alternatively, if we think, geometrically, of a Riemann sphere with a diameter of 1, then even allowing for continuous change such that a continuous quantity $z \in \mathbb{C}$ on the complex place is $\|z\| \to \infty$, the north pole, acting as a terminus of stereographic projection of the

Riemann sphere, corresponds to the discrete quantity 1, and any partition thereof is inconceivable. A partition of infinity is thus inconceivable using current Riemann spheres. Moreover, in physics, division by zero is regarded as an operation with no relation to physical phenomena, and zero distance is regarded as not being a physical quantity.

Nonetheless, we believe that the problem of the disappearance of matter information within the event horizon, and the problem of the partition of infinity, can be explained using the extended Riemann sphere.

We postulate that in the extended Riemann sphere, a converse Riemann sphere and inverse Riemann sphere are present as anti-Riemann spheres. The south pole of the converse Riemann sphere is connected to the north pole of the Riemann sphere, and the polarity of the converse Riemann sphere is opposite that of the Riemann sphere. In other words, in the converse Riemann sphere, it is the south pole (anti-north pole) that corresponds to infinity (anti-infinity), and the north pole (anti-south pole) that corresponds to 0 (anti-0). Thus, if we imagine an axis connecting the north pole of the Riemann sphere and the anti-north pole of the converse Riemann sphere on a straight line, that axis will represent infinity at the Riemann sphere terminus (north pole), and anti-infinity at the converse Riemann sphere terminus (anti-north pole). Meanwhile, the anti-south pole surface of the inverse Riemann sphere is connected on its obverse to the north pole of the Riemann sphere, and the polarity of the inverse Riemann sphere is flipped relative to that of the Riemann sphere on its obverse. For example, in the inverse Riemann sphere on the reverse side of the converse Riemann sphere, it is the anti-south pole surface that corresponds to 0 and the anti-north pole that corresponds to infinity. In the inverse Riemann sphere, the axis connecting the anti-south pole and anti-north pole enjoys a degree of freedom in regard to its rotation direction. Thus, on the above-described axis connecting the north pole of the Riemann sphere and anti-north pole of the converse Riemann sphere, the termini of the two spheres represent infinity and anti-infinity, and in addition, the interval between those termini is a slider along which infinity moves.

In the extended Riemann sphere, emergence can be defined as the phenomenon whereby an infinity formed in a null set (KU) of the positive world, and an anti-infinity formed in an anti-null set (anti-KU) of the anti-world, overlap and produce 1. In other words, if the diameter of the Riemann sphere in the positive world is 1, then this 1 can be defined as a discrete quantity on the anti-Riemann sphere in the anti-world after emergence. Alternatively, if, on a larger scale, the length of the above-described axis connecting the north pole and anti-north pole is 1, then as in the superposition operation of Table 1, this 1 can be interpreted as a post-emergence discrete quantity. In such case, in the Riemann sphere, time flows in a forward direction and the function undergoes addition via Σ towards infinity at the north pole, and in the converse Riemann sphere, time flows in a backward direction and the function undergoes addition via Σ towards the infinitesimal at the anti-south pole. If the superposition operation is executed with attention to the fact that the sum up to the infinite of a function and the sum up to the infinitesimal of the function differ depending on whether they are continuous quantities or discrete quantities, the superposition operation on time in the forward direction and the backward direction will result in the disappearance of time and the overlap of the infinite and zero occurs mathematically. The distance of the infinity superposed in this superposition operation will be the diameter of (length across) the extended Riemann sphere, and the 1 that is the discrete quantity arising after emergence due to the cut operation will correspond to this diameter. In other words, what becomes 1 after emergence differs depending on vantage point. Moreover, the post-emergence 1 is partitioned by the cut operation in the anti-world and gives rise to concepts in the anti-world. The concepts thereof are, for example, the extended complex plane of the anti-world (obtained by adding the ideal point to the complex plane) and the function represented thereon.

Thus, the problem of the disappearance of matter information in the event horizon can be solved by interpreting the collapsed matter information not as returning to the Riemann sphere, but as being translated through emergence to the anti-Riemann sphere. Likewise, the problem of the partition of infinity can be solved by interpreting the 1 that is an emergent discrete quantity as generating concepts through the cut operation in the anti-Riemann sphere.

In general relativity theory, the distance near the singularity resulting from division by zero is interpreted as being infinitely extended. Thus, if we consider this in terms of falling matter in the extended complex plane of the Riemann sphere, matter having a center of gravity at the ideal point would fall toward a 0 far from infinity. The interpretation that the information of matter that had fallen to 0 would radiate backwards over an infinite distance is unnatural. However, if we consider this in terms of falling matter in the Riemann spheres comprising the extended Riemann sphere, the distance to 0 from the singularity resulting from division by zero can be deemed to be 1 on the basis of the diameter of the Riemann sphere. Matter having a center of gravity at infinity (north pole) in the Riemann sphere falls to 0, which is separated by a distance of precisely 1, and falls to anti-infinity (anti-north pole) in the converse Riemann sphere, which is separated from 0 by a distance of precisely 1. A black hole in the Riemann sphere and white hole in the converse Riemann sphere will compete back-to-back with the Schwarzschild radius as a boundary between them, and, as in a Möbius strip or Klein bottle, will undergo attribute inversion and thereby connect infinity to anti-infinity. There will be a gap at the connection resulting from attribute inversion; KU will be the boundary with the connection on the infinity side; anti-KU will be the boundary with the connection on the anti-infinity side; and MU will be the center of the gap. Matter at infinity on the Riemann sphere will pass through the black hole and the white hole and then be reborn on the anti-Riemann sphere as the set concept, space-time, and function having anti-infinity as the sum. As a result of the foregoing, general relativity and quantum theory can be reconciled, and as a result of the cut operation, dynamization operation, and superposition operation, numbers, volumes, and concepts can be handled naturally within emergence phenomena. Also, the extended Riemann sphere allows *not* and *rev* to be handled explicitly. Furthermore, as detailed below, the observer problem of quantum theory can also be solved.

It bears noting that the symbol ♀ in the dynamization operation and superposition operation was first introduced by the lead author in his 2006 doctoral dissertation.[13] Further, the definitions of the dynamization operation and superposition operation that use ♀ were first published by the lead author et al. in DHU JOURNAL in 2020.[4] The same symbol was also used in the anti-Einstein field hypothesis (Some hypothesis to derive an anti-Einstein field).[3] The “anti” of the anti-Einstein field hypothesis is anti in the sense explained in Section 1.1, and means the anti of attribute inversion. In other words, it is synonymous with the anti of anti-Riemann sphere (anti-Riemann field). Thus, it is not an anti that negates something, in the sense of “anti-establishment” or “anti-nuclear”, and so is not a negation of relativity theory. Rather, in relativity theory, the anti-field plays the role of reinforcing the positive field with respect to the singularity.

Current quantum bits are represented using a Bloch sphere, but the quantum bits of the proposed method are represented using the extended Riemann sphere. This is illustrated in Figure 5.

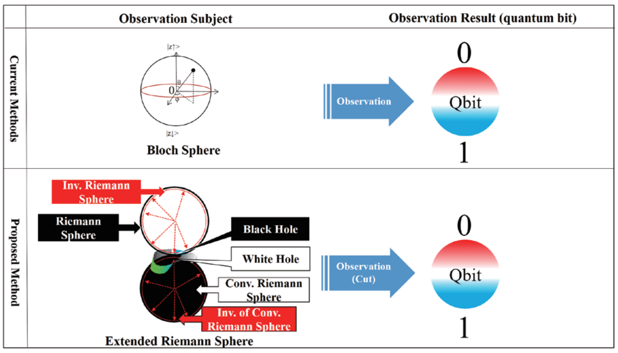

Figure 5: Difference of observation targets in current methods and proposed method

3.2 Basic Design of Emergent Quantum Gate

3.2.1 Overview

Simultaneous calculation of continuous quantity and discrete quantity cannot be envisioned with current quantum bits, where the result of simultaneous calculation of a continuous quantity interpreted as a discrete quantity. Therefore, the emergence of a continuous quantity from a discrete quantity cannot be envisioned, nor can the fact that the continuous quantity becomes a discrete quantity at the limit. Current quantum bits have only limited capability in this regard.

However, simultaneous calculation of continuous quantities and discrete quantities can be envisioned with quantum bits represented using the extended Riemann sphere. The superposition of infinity, which is the limit of the continuous quantity, and 0 (0≡∞) occurs in the anti-world, and as a result, the discrete quantity 1 emerges and a continuous quantity having anti-infinity as a sum is born in the positive world. In other words, emergence in the anti-world creates the discrete quantity 1, and the 1 that appears in the anti-world is partitioned in the positive world, giving rise to a continuous quantity. This simultaneous calculation of continuous quantity and discrete quantity is illustrated in Figure 6. Simultaneous calculation of continuous quantity and discrete quantity is as described in DHU JOURNAL, vol. 06, 2019, p. 10. We have named this analog-and-digital simultaneous calculation “quantal calculation”.

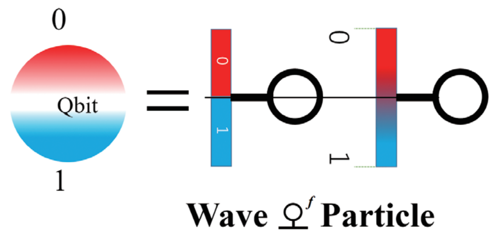

Figure 6: Explanation of quantal calculation

In quantal calculation, quantum bits behave like oracles in computability theory, and output time-dependent wave functions or the like. In other words, emergence in the anti-world gives rise to an “oracle” in the positive world. Conventional quantum gates, which handle only relative phase of spin in Bloch spheres, cannot become oracles, but quantum gates that handle relative amplitude in the extended Riemann sphere can become oracles through the connection of infinity and anti-infinity. Thus, we have undertaken experiments in which quantum superposition states are controlled using lasers, to confirm emergence phenomena in the anti-world.

3.2.2 Nature of Emergence

The essence of the concept output by quantum gate emergence is made clear by using the inversion operation to analyze the interval between positive and anti. Thus, the premise (p’→q’) is divided by cut operation into the thesis (p→q) and its inverse (not·p→not·q), and that which exists between the thesis and the inverse is analyzed using the inversion operation.

When the cut operation is performed two-dimensionally, (p’→q’)*cut*2=(*p*→*q*)♀(not·p→not·q), and if inv·(p’→q’)=(not·p→not·q), then rev·(p’→q’)=0. In other words, the obverse is (*p*→*q*), *inv* is (not·p→not·q), and there is no *rev*, as indicated in Figure 2.

When the cut operation is performed three-dimensionally, (p’→q’)*cut*3=(*p*→*q*)♀X♀(not·p→not·q), and if inv·(p’→q’)=(not·p→not·q), then rev·(p’→q’)=*X*. In other words, the obverse is (*p*→*q*), *inv* is (not·p→not·q), and *rev* is rev·(p’→q’)=*X*.

When the cut operation is performed in infinite dimensions, (p’→q’)*cut*∞=(*p*→*q*)♀Z♀(not·p→not·q), $Z=Z_1$♀Z_2♀…♀Z_∞, and rev·(p’→q’)=*Z* is understood to represent the nature of emergence in the extended Riemann sphere. Thus, we formulated rev·(p’→q’)=*Z* and attempted an interpretation.

In a state where the dynamization operation has continuity, rev·(p’→q’)=*Z* was interpreted as follows using a Riemann sphere.

$$|Z|^2 = \left|\lim_{n\to\infty}\right|^2 = \lim_{n\to\infty}\int_0^\infty |a_n(x)N^n(x)|^2 dx = \infty.$$

In the Riemann sphere, Z can be considered to represent a nature where a concept is attracted to and continuously superposed on the north pole (infinity), and it is thus predicted that rev·(p’→q’)=*Z* is a nature where gravity is produced after emergence.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = (p \to q)\ \text{오}\ (\lim_{n\to\infty})\ \text{오}\ (not \cdot p \to not \cdot q).$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') = \text{"nature of gravity"}.$$

In addition rev·(p’→q’)=*Z* was interpreted as follows using a converse Riemann sphere.

$$|Z|^2 = \left|\lim_{n\to 0}\right|^2 = \lim_{n\to 0}\int_{-\infty}^{0} |a_n(x)N^n(x)|^2 dx = anti \cdot 0\,.$$

When interpreted using the converse Riemann sphere, Z represents a nature where a concept is attracted to and continuously superposed on the anti-south pole (infinitesimal) as a continuous quantity, and it is thus predicted that rev·(p’→q’)=*Z* is a nature where the action of a fundamental force other than gravity is produced after emergence.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = (p \to q)\ \text{오}\ \left(\lim_{n\to 0}\right)\ \text{오}\ (not \cdot p \to not \cdot q).$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') =$$

$$\text{"nature of electromagnetic force, strong force, weak force"}.$$

In a state where the dynamization operation lacks continuity due to discrete quantification, rev·(p’→q’)=*Z* was interpreted as follows using a Riemann sphere.

$$|Z|^2 = \left|\lim_{n\to\infty}\right|^2 = \lim_{n\to\infty}\sum_{i=0}^{\infty}\left|a_{n,i}N^{n,i}\right|^2 = \infty.$$

In the Riemann sphere, because a discrete quantity is superposed at the north pole (infinity), Z can be considered to represent the nature of a discrete quantity. Thus, if the nature of a discrete quantity is passed on after emergence, then $\infty \to a_nN^n + a_{n+1}N^{n+1} + \ldots + a_\infty N^\infty$ - in other words, Z can be formulated as a nature where infinity is divided in infinite dimensions and a discrete quantity is produced after emergence.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = (p \to q) + \left(\lim_{n\to\infty}\right) + (not \cdot p \to not \cdot q).$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}')$$

$$= \text{"nature where discrete quantity emerges in infinite dimensions"}$$

In addition, rev·(p’→q’)=*Z* was interpreted as follows using a converse Riemann sphere.

$$|Z|^2 = \left|\lim_{n\to 0}\right|^2 = \lim_{n\to 0}\sum_{i=-\infty}^{0}\left|a_{n,i}N^{n,i}\right|^2 = anti \cdot 0.$$

In the converse Riemann sphere, because a discrete quantity is superposed at the anti-south pole (infinitesimal), here as well Z can be viewed as representing the nature of a discrete quantity. Thus, if the nature of a discrete quantity is passed on after emergence, then $0 \to a_nN^n + a_{n-1}N^{n-1} + \ldots + a_0N^0$ – in other words, Z can be formulated as the nature where 0 is divided in finite dimensions and a discrete quantity is produced after emergence.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = (p \to q) + \left(\lim_{n\to 0}\right) + (not \cdot p \to not \cdot q).$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}')$$

$$= \text{"a nature where discrete quantity emerges in finite dimensions"}$$

Thus, the laser experiment described above may be able to discern emergent phenomena in even smaller space-time than the following equation we proposed.

$$(\emptyset\text{: anti-}\emptyset)\ \text{오}^{f}\ (\infty + \text{anti-}\infty)$$

$$\mathrm{KU} = \lim_{1\to 0}\ \text{오}\lim_{0\to 1}$$

Incidentally, in the inverse Riemann sphere, the inverse is not conceived in terms of *not* as in logic, and instead, can be conceived in terms of *rev* as in the novel calculation standards. Consequently, the inverse Riemann sphere can be observed using *rev* as well. In other words, to use the Earth as a metaphor as in Figure 7, the *inv*-Earth, and not the *not*-Earth, can be observed using *rev*.

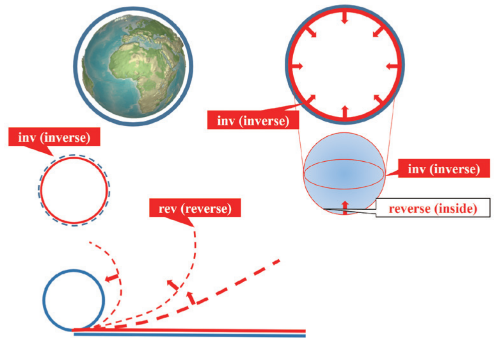

Figure 7: Relationship between rev of novel computation standards and inverse Riemann sphere

In the extended Riemann sphere, orientation can be fixed to the obverse (foreside) or reverse (inside) of the sphere. If obverse and reverse surface orientations are represented as vectors, the *shiki* terms represented in base vector decomposition can be divided between obverse (foreside) and reverse (*rev*) using the element representing orientation. Of these, the reverse (*rev*) is calculated using an inversion operation.

The nature of an emergence is observed using the inverse Riemann sphere (*inv*). The inverse Riemann sphere is a sphere in which energy is present somewhere between the center and the sphere surface. The sphere surface is the anti-south pole surface, and the center is the anti-north pole. Thus, the energy is present between positive and anti. By analyzing the energy present between positive and anti with an inversion operation, it is possible to observe the nature of an emergence. Thus, if the obverse Riemann sphere is (*p*→*q*), (*p*→*q*)cut ∞ = sphere surface 오 Z 오 center, where $Z = Z_1$ 오 Z_2 오…오 Z_∞, rev·(p’→q’)=*Z* is observed.

The following formula can be interpreted for the state where the dynamization operation has continuity.

$$|Z|^2 = \left|\lim_{n\to 0}\right|^2 = \lim_{n\to 0}\int_{\infty}^{0} |a_n(x)N^n(x)|^2 dx = anti \cdot 0.$$

When this Z is interpreted using the inverse Riemann sphere, a first possibility is conceivable in which, when the thesis (p’→q’) is subjected to a cut operation using *inv*, attracted to the anti-north pole (infinitesimal), and superposed thereon while shrinking, the mass superposed as a result of such shrinking is rev·(p’→q’)=*Z*. This mass shrunk to the infinitesimal is referred to as “shrink mass”. Meanwhile, there is also a second possibility in which a continuous quantity is Z.

In light of these two possibilities, it is feasible to think that, when the infinitesimal emerges ubiquitously as a concept in the extended complex plane of the anti-world, a shrink mass having a continuous quantity appears as the nature of such infinitesimal, but is oriented toward growth in the anti-world because of attribute inversion.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = surface\ \text{오}\ (\lim_{n\to 0})\ \text{오}\ \text{center}$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') = \text{"shrink mass"}$$

Alternatively, the following formula can also be interpreted for the state where the dynamization operation has continuity.

$$|Z|^2 = \left|\lim_{n\to\infty}\right|^2 = \lim_{n\to\infty} \int_0^\infty |a_n(x)N^n(x)|^2 dx = anti \cdot \infty.$$

When this Z is interpreted using the inverse of converse Riemann sphere, a first possibility is conceivable in which, when the thesis (p' →q') is subjected to a cut operation using *inv*, attracted to the anti-north pole (infinity), and superposed thereon while being concentrated, the mass superposed as a result of such concentration is rev·(p' →q')=*Z*. This mass concentrated at infinity is referred to as "growth mass". Meanwhile, there is also a second possibility in which a continuous quantity is Z. In light of these two possibilities, it is feasible to think that, when the infinite emerges as a concept on the extended complex plane of the positive world, a growth mass having a continuous quantity appears as the nature of such infinite, but is oriented toward shrinking in the positive world because of attribute inversion.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = surface\ \text{오}\ \left(\lim_{n\to 0}\right)\ \text{오}\ \text{center.}$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') = \text{"growth mass".}$$

The following formula can be interpreted for the state where the dynamization operation lacks continuity due to discrete quantification.

$$|Z|^2 = \left|\lim_{n\to 0}\right|^2 = \lim_{n\to 0} \sum_{i=-\infty}^{0} \left|a_{n,i}N^{n,i}\right|^2 = anti \cdot 0.$$

When this Z is interpreted using the inverse Riemann sphere, a state occurs in which, when the thesis (p' →q') is subjected to a cut operation using *inv*, attracted to the anti-north pole (0), and superposed thereon while shrinking, this shrinking pushes infinity to the smallest possible discrete number. This state in which infinity is pushed to the smallest possible discrete number is referred to as "growth density". This growth density can be considered to represent a nature where energy is not inverted at the time of emergence because it is not attribute-dependent, and the infinitesimal begins to grow after emerging ubiquitously as a concept on the extended complex plane of the positive world.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = (p \to q) + \left(\lim_{n\to 0}\right) + 0.$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') = \text{"growth density".}$$

Alternatively, the following formula can also be interpreted for the state where the dynamization operation lacks continuity due to discrete quantification.

$$|Z|^2 = \left|\lim_{n\to\infty}\right|^2 = \lim_{n\to\infty} \sum_{i=0}^{\infty} \left|a_{n,i}N^{n,i}\right|^2 = anti \cdot \infty.$$

When this Z is interpreted using the inverse of converse Riemann sphere, a state occurs in which, when all concepts are attracted to the anti-north pole (infinity) and superposed thereon while being concentrated, the smallest possible discrete number is pushed to infinity as a result of such concentration. This state in which the smallest possible discrete number is pushed to infinity is referred to as "shrink density". This shrink density can be considered to represent a nature where energy is not inverted at the time of emergence because it is not attribute-dependent, and infinity begins to shrink after emerging as a concept in the extended complex plane of the anti-world.

$$(\mathrm{p}' \to \mathrm{q}')\ cut\ \infty = north\,pole + \left(\lim_{n\to\infty}\right) + \text{origin point.}$$

$$\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}') = \text{"shrink density".}$$

Moreover, based on the observations in this section, it is theoretically possible to conceive of antigravity and distant gravity. To begin, antigravity can be represented by the following formula by defining the nature of gravity as having been inverted.

$$\mathrm{anti} \cdot \text{"nature of gravity"} = \mathrm{anti} \cdot \{\mathrm{rev} \cdot (\mathrm{p}' \to \mathrm{q}')\}.$$

The inversion (anti) of nature occurs in the converse, as was exemplified using α and β in section 2.3. Thus, when the operation (con) for calculating a converse Riemann sphere corresponding to the converse is used to reverse the nature of gravity and assign a pole at the point where antigravity is controlled in each of the proposition variables p and q, the result is as follows.

$$con \cdot (\mathrm{S}' \to \mathrm{N}')\ cut\ \infty$$

$$= con \cdot (S \to N)\ \text{오}\ (\lim_{n\to\infty})\ \text{오}\ \mathrm{con} \cdot (not \cdot S \to not \cdot N).$$

$$\text{Therefore, } \mathrm{anti} \cdot \{\mathrm{rev} \cdot (\mathrm{S}' \to \mathrm{N}')\}$$

$$= \mathrm{rev} \cdot \{\mathrm{con} \cdot (\mathrm{S}' \to \mathrm{N}')\} = \left(\lim_{n\to\infty}\right) = \mathrm{anti} \cdot \text{nature of gravity.}$$

Next, the distant gravity for controlling antigravity can be represented by the densities of the following formulae.

$$con \cdot \infty + \left(\lim_{n\to\infty}\right) + con \cdot origin\ point = anti \cdot gravity\ shrink\ density.$$

$$con \cdot (S \to N) + \left(\lim_{n\to 0}\right) + con \cdot 0 = anti \cdot gravity\ growth\ density.$$

3.2.3 Nature of Quanta

It can be considered that the quantum bits in current methods handle the states of quantum gates using cut operations. Thus, because of observation, the final state is fixed at 0 or 1 and an inversion operation is inconceivable, as shown in Figure 8. The quantum state is therefore (S→N) cut2=a_0|0⟩+a_1|1⟩, which contains no *rev* information.

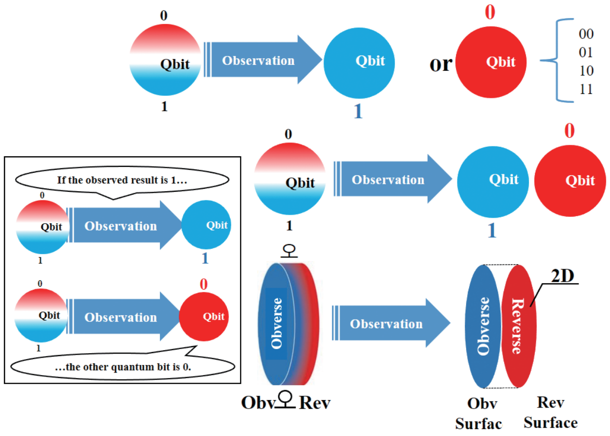

Figure 8: Current quantum bits

With the quantum bits of the proposed method, the state of a quantum gate is handled by cut operation, dynamization operation, superposition operation, and inversion operation. Thus, the *rev* of quanta is handled as in the following formula.

$$(S \rightarrow \mathrm{N})\ cut\ 2 = a|0\rangle \text{오} \mathrm{b}|1\rangle.$$
$$X = rev \cdot (S \rightarrow \mathrm{N}) = rev\ common\ to\ obverse\ and\ reverse$$

$$(S \rightarrow \mathrm{N})\ cut\ 2 = a|0\rangle + \mathrm{b}|1\rangle.$$
$$X = rev \cdot (S \rightarrow \mathrm{N}) = rev\ distinct\ between\ obverse\ and\ reverse$$

In quantal calculation, a quantum state containing rev information is named a “quantal state”.

Current methods are limited in that, because the state can only be interpreted digitally, the analysis must begin and end with the discrete numbers 0 and 1. However, in the proposed method, in addition to handling quanta as waves as in a|0⟩오 b|1⟩, the quality of such quanta can be handled using *rev*. Thus, in the proposed method, spin can be conceived not only as |0⟩ and |1⟩, but as a |↑⟩ oriented in any direction and its opposed |↓⟩. In such case, the *rev* of quanta is expressed by the following formula.

$$(S \rightarrow \mathrm{N})\ cut\ 2 = a|\uparrow\rangle \text{오} \mathrm{b}|\downarrow\rangle.$$
$$X = rev \cdot (S \rightarrow \mathrm{N}) = rev\ \ common\ to\ obverse\ and\ reverse$$

$$(S \rightarrow \mathrm{N})\ cut\ 2 = a|\uparrow\rangle + \mathrm{b}|\downarrow\rangle.$$
$$X = rev \cdot (S \rightarrow \mathrm{N}) = rev\ distinct\ between\ obverse\ and\ reverse$$

In such case, when *rev* is considered with respect to (*S*→N) *cut* 3, a “오” connected to the same infinity as the intermediate term Z of (*S*→N) *cut* ∞ appears. The 오 at this infinity can be interpreted as a term related to emergence in the extended Riemann sphere as shown in 3.2.2 “Nature of Emergence”. Therefore, an extended Riemann sphere constructed by geometric expansion in logic could be the foundation of a novel quantum logic calculation system.

3.2.4 Mathematical Structure of Quanta

In current quantum computing, quantum states are generally represented using Bloch spheres,[11] but representation using Riemann spheres[12] is also a possibility. The proposed method uses the extended Riemann sphere.

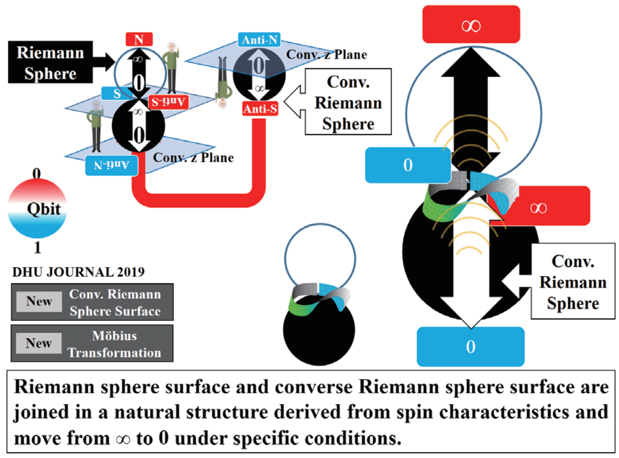

Figure 9: Representation of attribute inversion using extended Riemann sphere

As shown in Figure 9, the extended Riemann sphere is connected at the pole at which a Riemann sphere that has undergone attribute inversion. For example, the south pole (S) of a Riemann sphere is connected to the anti-south pole (anti-S) of a converse Riemann sphere that has been rotated 180 degrees. The extended Riemann sphere has been defined so as to correspond to a logical proposition. For example, the Riemann sphere corresponds to the thesis (*P*→*Q*), and the converse Riemann sphere corresponds to its converse (*Q*→*P*).

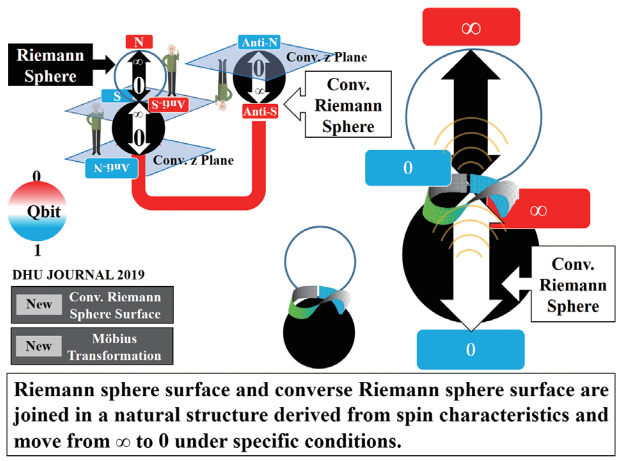

Figure 10: Connection of poles where attribute inversion occurs

As illustrated in Figure 10, on the converse Riemann sphere surface, stereographic projection is defined from the anti-south pole with respect to the extended complex plane where such anti-north pole is grounded. If the Riemann sphere surface is taken as a reference, then attribute inversion results in the anti-north pole of the converse Riemann sphere surface being anti-infinity, and the anti-south pole being the anti-origin point. Conversely, if the converse Riemann sphere is taken as a reference, then the anti-north pole can be regarded as the south pole and the anti-south pole can be regarded as the north pole. In other words, in general, a Möbius transformation defined on a Riemann sphere surface can be used as-is in the converse Riemann sphere by inverting the reference. That is to say, in the extended Riemann sphere, quantum gate control can be described with a Möbius transformation, as in the case of the Riemann sphere surface.

3.2.5 Mathematical Structure of Emergence

In a Möbius strip, an obverse surface and reverse surface that are connected to each other by attribute inversion are connected, but in the proposed extended Riemann sphere, an infinity and anti-infinity that are related to each other by attribute inversion are connected. This section will clarify the mathematical structure of Möbius strips and extended Riemann spheres.

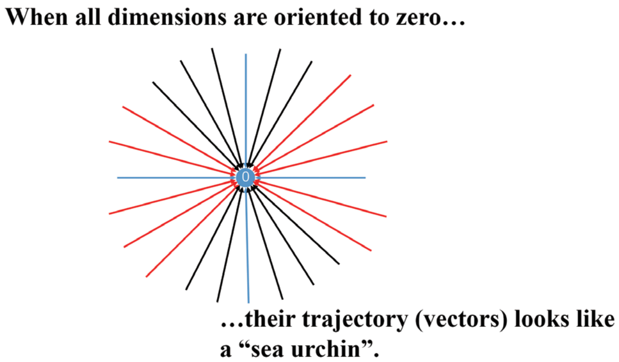

Figure 11: State of infinite dimensional convergence on origin point 0

Figure 11 concisely illustrates a state where, with respect to a *shiki* represented by an infinite-dimensional orthogonal basis, all terms are oriented to an origin point. In this state, the trajectory traced by each term of the *shiki* can be sketched like a "sea urchin" centered on the origin point 0.

If, in order to flesh out the picture of *shiki* converging on origin point 0, we were to imagine a spatial coordinate (*shiki*) represented by a three-dimensional orthogonal basis, the convergence of *shiki* in the fourth and subsequent dimensions would appear impossible. This would be because, as shown in Figure 12, there would be no room left to add the fourth and subsequent orthogonal bases.

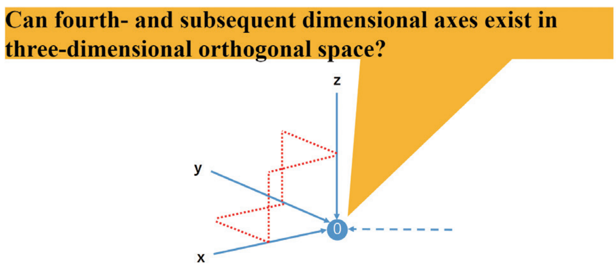

Figure 12: Room for orthogonal basis in three-dimensional orthogonal space

Thus, for black holes, a theory in which the fourth and subsequent dimensions converge while crossing orthogonally at origin point 0 is needed. In other words, a theory that realizes a condition such as the space-time-swallowing black holes described by Prof. Hawking and others is necessary.

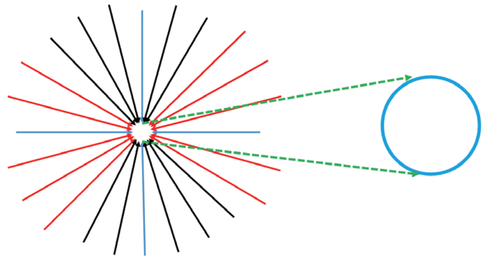

Figure 13: Infinitesimal gap

As shown in Figure 13, a limit space is present as a gap at the center of the sea urchin. This gap would be, in outer space, an entity the size of the Schwarzschild radius in a black hole, and, in infinitesimal space, an entity no larger than the smallest unit (infinitesimal) that can be attained by a space of roughly the Planck length.

Thus, as an entity determining the inside of the infinitesimal, a string such as a rubber band surrounded by the gap is expressed as emptiness (KU). Furthermore, as in Figure 14, the inside (*rev*) of KU is expressed as nothingness (MU). As a result of the directionality of convergence, the orientation of MU is defined by a vector oriented to the center of KU.

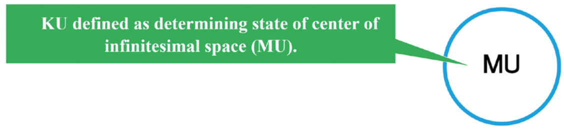

Figure 14: Relationship between KU and MU

Based on this definition, MU is also *shiki* and can be sketched using a dynamization operation as shown in Figure 15. ⚲ is drawn like a control lever because the pole connecting the Riemann spheres is a control point and KU is present at this control point.

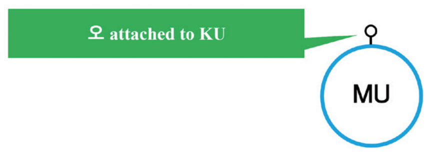

Figure 15: Relationship between MU and ⚲

At the pole, the infinity of the obverse is connected to the anti-infinity of the anti, and it thus appears that there is a breakpoint (gap) in the KU string. Therefore, it is clear, as in Figure 16, that where the KU string is broken at a certain point, infinity is connected to infinity, forming a single loop, and after the break, a ⚲ of which both ends approach infinity can be sketched.

Figure 16: Existence of break point on KU

Thus, in order to connect the infinity of the obverse and the anti-infinity of the anti, as shown in Figure 17, one side of

♀ is twisted by 180 degrees and connected to MU. In other words, if infinity at ♀ is the obverse, its inside (*rev*) is MU. In addition, the 180-degree twisting deformation is represented by the function f, in which the ♀ with added deformation is represented by $♀^f$.

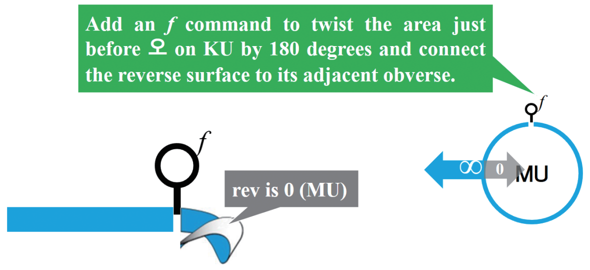

Figure 17: Connection between infinity and MU produced by $♀^f$

Therefore, when a 180-degree rotation is imparted to one ♀ using the function f, KU is as depicted in Figure 18. More specifically, through the function f, the ∞ of the obverse and the MU of the *rev*-obverse (reverse) are connected ($0 \equiv \infty$), forming a Möbius strip. In DHU JOURNAL in 2019,[3] this Möbius strip was explained using a Klein bottle and represented as a matrix.

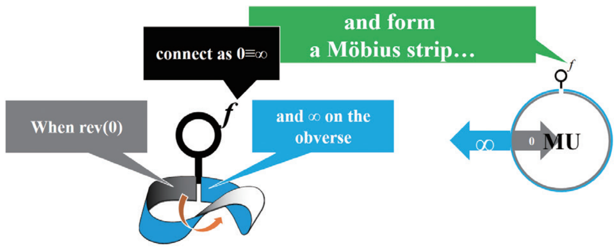

Figure 18: Formation of Möbius strip $♀^f$ by function f

The Möbius strip $♀^f$ formed by the function f can be used to construct a theory in which four or more dimensions converge while crossing orthogonally at origin point 0. This is because: first of all, the smooth twist added to the strip makes it possible to have a vector at infinity which crosses the strip orthogonally, as shown in Figure 19; and second of all, inverting and connecting zero and infinity just below ♀ allows a vector that orthogonally crosses KU, *i.e.*, Möbius strip, to be present at infinity. In other words, it is possible for all vectors to cross orthogonally in an infinitesimally small state, both at micro and at macro levels. The *rev* of KU is MU in the form of absolute nothingness. As a result of emergence, the infinity on the obverse changes into the anti-infinity on the reverse (converse Riemann sphere) after passing through the gap between infinity and MU of ♀ and anti-♀. On the anti, it appears as though nothingness is translated to anti-infinity and being has emerged from nothingness. The foregoing is the mathematical structure of emergence at the connection of poles where attribute inversion has occurred.

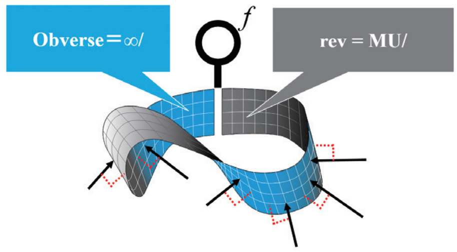

Figure 19: Möbius strip $♀^f$, which realizes orthogonal crossing at the infinitesimal

Current quantum gates based on Bloch spheres and Riemann spheres are limited in that they do not realize attribute inversion at poles, and thus, to be candid, are incapable of using anything other than |0⟩ and |1⟩ to control the entanglement and superposition of quanta. Quantum gates based on the extended Riemann sphere remove this limitation.

4. Comparison with Current Methods

We will now discuss the differences between the Bloch spheres[11] and Riemann spheres[12] which are the basis of current methods and the extended Riemann sphere which is the basis of our proposed method.

4.1 Unnaturalness of Spinor Representation Using Bloch Spheres

Spinor representation in Bloch spheres has an unnatural aspect – namely, the relative phase restrictions set to ensure there are no contradictions. For example, Bloch sphere spinor representation can be expressed using the following formula.

$$
\begin{aligned}
\psi > \; &= a'\,|z\uparrow> \; + b'|z\downarrow> = ae^{i\theta_1}\left|z\uparrow> \; + be^{i\theta_2}\right|z\downarrow> \\
&= e^{i\theta_1}\left(a\left|z\uparrow> \; + be^{i(\theta_1-\theta_2)}\right|z\downarrow>\right) \\
&= e^{i\theta_1}\left(\cos\theta'\left|z\uparrow> \; + \sin\theta'\, e^{i(\theta_1-\theta_2)}\right|z\downarrow>\right) \\
&= e^{i\delta}\left(\cos\theta'\left|z\uparrow> \; + e^{i\phi'}\sin\theta'\right|z\downarrow>\right) \\
&= e^{i\delta}\left(\cos\frac{\theta}{2}\left|z\uparrow> \; + e^{i\phi}\sin\frac{\theta}{2}\right|z\downarrow>\right)
\end{aligned}
$$

With respect to the relative phase, as a result of the following restriction, $z\uparrow>$ is $z\downarrow>$ when $\theta=\pi$.

$$0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi$$

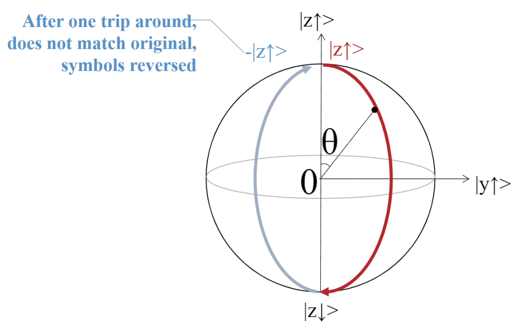

Figure 20: Spinor representation using Bloch spheres

As shown in Figure 20, when $\theta=\pi$, $z\uparrow>$ becomes $z\downarrow>$, matching the figure, but when $\theta=2\pi$, the symbols reverse and do not match the figure. Meanwhile, in a Möbius strip where the symbols reverse after one trip around the strip, this reversal of symbols can be expressed naturally, as shown in Figure 21.

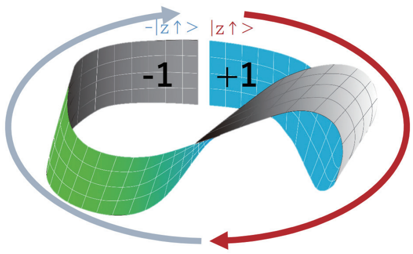

Figure 21: Spinor representation using Möbius strip

4.2 Unnaturalness of Spinor Representation Using Riemann Sphere Surfaces

Spinor representation using Riemann sphere surfaces also has an unnatural aspect. To confirm this unnaturalness, let us first define the Möbius transformation M_g using the following formula.

$$M_g\left(\frac{z_1}{z_2}\right)=\frac{r_1}{r_2}e^{i\,(\theta_1-\theta_2)}=re^{i\theta}$$

When transformation in a spinor representation is defined using this Möbius transformation, M_g, M_g can be treated as a rotation operation on the Riemann sphere, as in Figure 22.

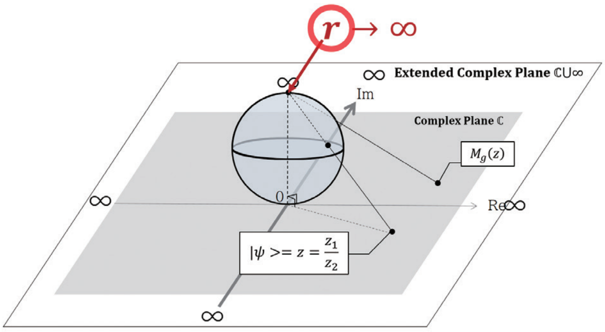

Figure 22: Rotation operation on Riemann sphere surface by Möbius transformation

For relative amplitude $r=r_1/r_2$ as well, the rotation operation on the Riemann sphere surface is represented by the Möbius transformation M_g. As shown in Figure 23, we can see that relative phase(θ_1-θ_2) matches the Möbius strip, but for relative amplitude, there is nothing beyond relative amplitude r, and thus, unnaturally, closure does not occur.

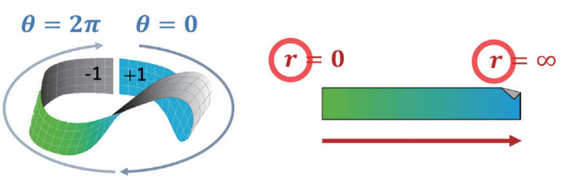

Figure 23: Difference between relative phase (left) and relative amplitude (right) in Riemann spheres

For example, in cases like that shown in Figure 24, there is nothing beyond where relative amplitude undergoes attribute inversion and is connected at infinity.

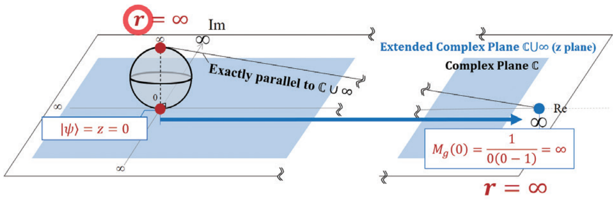

Figure 24: When z=0 in $M_g(z)=\frac{1}{z(z-1)}$

4.3 Resolving Unnaturalness with Extended Riemann sphere

Therefore, the extended Riemann sphere operates according to "*obverse* ∪ *converse* ∪ *inverse* ∪ *contrapositive*" on the basis of the proposed method, and when connected as in Figure 25, resolves the unnaturalness of spinor representation in Bloch spheres and Riemann spheres.

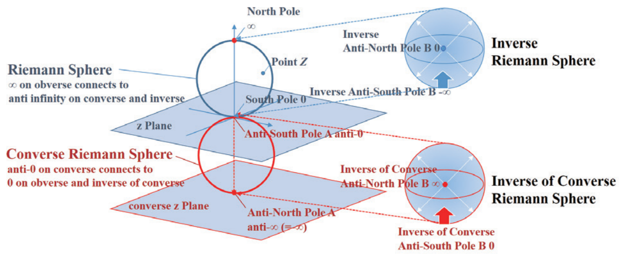

Figure 25: Connection of poles in extended Riemann sphere

For reference, the quantum gate mathematical structure we have applied to patent through the University of Tokyo is shown in Figure 26.

Novel Gate	PPT / CON / INV / REV / ANT / CTP / MU / MGN+ / MGN- / KU			
Function	Converse Riemann sphere/inverse Riemann sphere connection method (expansion in amplitude direction of wave function), etc.			
Currently-Known Techniques	Quantum gate systems based on SU(2) unitary matrix	On Riemann sphere surface, Möbius group turns SU(2) unitary matrix into SO(3) rotation group	On Riemann sphere surface, a group is formed with respect to rotation but not with respect to amplitude	Quantum spin experiences attribute inversion after 1 rotation
	Change of wave function considering spin is expressed by U(2) unitary matrix			

Figure 26: Quantum gate mathematical structure

5. Experiment

It is necessary to execute division by zero and coordinate expression of infinity using ♀ calculations on current computers, using the above-described 2019 physical model[3] and 2020 mathematical model.[4]

As a prototype, in 1993, we completed a 3D model to clarify our overall image of quantum gates. Then, in 2010, we produced a working model having a ♀ dynamization operation as an operating principle. Further, in 2018, we did a trial run of a ♀f superposition operation calculation principle in a gyro model, and in 2019, we did a trial run of a ♀f resonance principle in a twin gyro model. These models are shown in Figure 27.

Figure 27: Experimental models having dynamization operation and superposition operation as operating principles

We now need to derive basic principles of the above-detailed quantum gates etc. from the actions of these models, verify the actions on computers, and develop new artificial nerve connection systems based on the artificial ego.

5.1 Implementation of Artificial Nerves

In the artificial nerves we have produced thus far, sigmoid functions have not been used for activation functions. Figure 28 summarizes the relationship between quantum bits, dynamization operations, superposition operations, and the extended Riemann sphere.

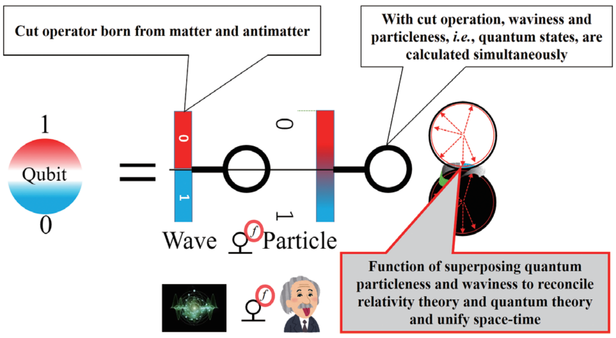

Figure 28: Relationship between quantum bits, dynamization operations, superposition operations, and extended Riemann sphere

We believe a neural connective structure can be represented using the extended Riemann sphere. Therefore, as was explained in "3. Emergent Quantum Gates", we envision a structure like that in Figure 29, in which the positive world and anti-world are connected as in a Möbius strip.

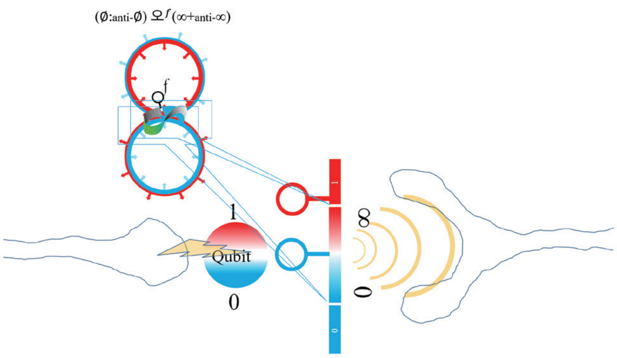

Figure 29: Neural connection model based on extended Riemann sphere

Thus, we have applied the operating principle of quantum gates based on the extended Riemann sphere and attempted to construct an emergence-imitating artificial nerve model on current computers.

As discussed above, division by zero and coordinate expression of infinity are possible using quantum gates based on the extended Riemann sphere, and connection between infinity and nothingness is possible using the shape of the Möbius strip shown in Figure 30. Thus, in our program, we have attempted to realize a process in which infinity and anti-infinity are connected automatically in division by zero, which conventionally has necessitated error processing.

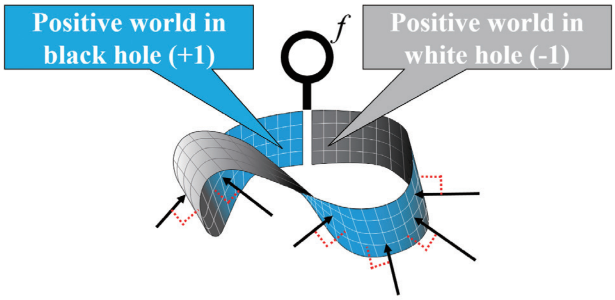

Figure 30: Connection between positive world and anti-world in quantum gates

5.2 Results of Implementation of Artificial Nerves

In this section, we confirm that, as a result of applying the operating principle of quantum gates based on the extended Riemann sphere to construct an emergence-imitating artificial nerve model on current computers, the strip that was not closed in the relative amplitude direction in Figure 23 was closed, like a Möbius strip, in the relative amplitude direction as in Figure 31.

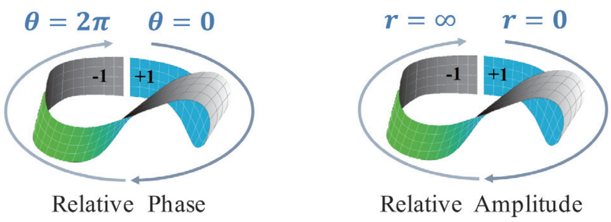

Figure 31: Behavior of relative phase and relative amplitude of constructed artificial nerve

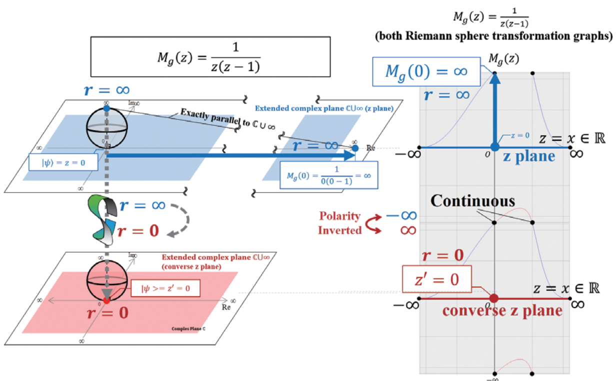

Figure 32: Specific behavior of constructed artificial nerve in experiment

With regard to the specific behavior in the experiment of the constructed artificial nerve, as shown in Figure 32, relative amplitude behavior was confirmed using the following formula, which in control engineering is considered a transfer function.

$$M_g(z) = \frac{1}{z(z-1)}$$

More specifically, we confirmed that, when $M_g(z)$ is used to translate the point z=0 or z=1 on complex plane $\mathbb{C}\cup\infty$ (z plane), assuming $z=re^{i\theta}$, r leaps from 0 to ∞.

In current coordinate displays such as logarithmic graphs, when r leaps to the ideal point, an error occurs and the program stops. However, we have confirmed that, in a coordinate display using the extended Riemann sphere, r can be handled as the north pole of the Riemann sphere, thus eliminating the need for error processing. The north pole of a Riemann sphere of radius 1 simply means "1" to a computer.

Further, we have confirmed that infinity and anti-infinity are connected, as in the graph shown on the right side in Figure 32. More specifically, if we assume the point where $r=\infty$ is translated to point 0 on another complex plane $\mathbb{C}\cup\infty$ (converse z plane), by using a generalized Riemann sphere transformation (mapping ∞ to 1), $M_g(z)$ can be handled as a continuous function even at an infinite point. As a result, the nature of $M_g(z)$ on the z plane can be passed on as $M_g(z)$ to the converse z plane after dispersal. Moreover, effects from the converse z plane will also appear in the z plane. This reciprocal interference, which can be seen as noise, can now be conceived as a basis for controlling the converse z plane from the z plane.

In light of the foregoing, in confirming the actions of the artificial nerves based on our proposed quantum theory model in which commonly used sigmoid functions are not used as activation functions, we have confirmed that division by zero and coordinate expression of infinity using the novel calculation standards can be reproduced in calculations by means of topological space transition. That is to say, when division by zero is performed, an infinity coordinate is exceeded in a different topological space (unconnected output side synapse) and a signal appears. At this time, if the input coordinate (synapse acting as input phase) is set to the blue area in Figure 32, a curve such as that in the graph appears as a continuous waveform. Then, the signal disappears when the infinite dimension $M_g(z)=\infty$ is reached, and in that instant the signal also appears, having maintained the same curve characteristic (function), at an output coordinate (synapse acting as output phase) in the red area on the right side of Figure 32. In other words, we have confirmed that, in an artificial nerve created using quantal principles, when a stimulus reaches the limit of the infinite, a phase transition occurs to 0, at which the stimulus disappears. Furthermore, we have confirmed that, in the coordinate expression of infinity in the artificial nerve, a monotonically increasing smooth transfer function that satisfies $f(0)=0$ in a form that diverges to $f(x)=\infty$ at $x=\infty$ and to $f(x)=-\infty$ at $x=-\infty$ will generate sigmoid functions. In other words, we have seen that, when the coordinate expression of infinity in the artificial nerve constructed in the experiment is used, diverse sigmoid functions can be generated automatically by means of a transfer function that diverges at the ideal point.

5.3 Discussion

The confirmation of a similarity between a neural firing rate and a sigmoid waveform is a widely-known fact in engineering fields. As a result, sigmoid functions are now used as activation functions in neural networks.

In the experiment, we implemented, on a current computer, an artificial nerve model based on a quantum theory model that should be referred to as a quantal connection model, and did not use sigmoid functions.

The results of the experiment confirmed that when division by zero, which hitherto has been neglected, ignored, and avoided in mathematics, physics, and computer science, is handled using 오 calculation signal processing, a signal appears that maintains continuity after the topological space transition, and a sigmoid waveform appears unintentionally, as shown in Figure 33.

While it is difficult to say that we have succeeded in recreating human neural connections, if nothing else, we have confirmed that analog control related to long-term memory and short-term memory in LTD/LTP connections, and neural ON/OFF digital signal processing, can be recreated simultaneously in a quantal connection. In this sense, it is undeniable that our experiment has been an important step forward in the development of the artificial ego.

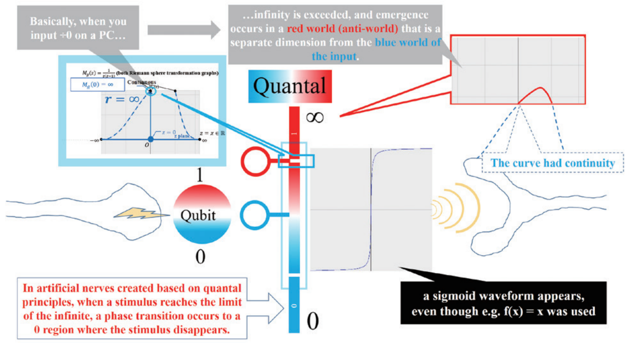

Figure 33: Discussion of results of experiment

6. Wrap-Up

While this may not be a quantum mind theory of a Nobel Prize winner in Physics, the manifestation of a natural sigmoid waveform in an emergence-imitating artificial nerve in which extended Riemann sphere-based quantum gates were implemented as operating principles is nonetheless an important discovery, in that it links black holes, white holes, emergence, and neural connections. We have shown that the engineering applications of the extended Riemann sphere, which reconciles relativity theory and quantum mechanics, do not lie in the distant future and will spark immediate evolution in the artificial nerve and artificial ego engineering fields.

7. Challenges Going Forward

With respect to applications in engineering fields, to achieve an artificial ego that emerges autonomously, it will be necessary to implement networks incorporating artificial nerves that take the extended Riemann sphere and novel calculation standards as an operating principle, and confirm their behavior.

With respect to applications in medical fields, medical schools have noted the feasibility of models that recreate stage gates for illnesses, and have begun research on them.

With respect to applications in physics fields, because our theory predicts the existence of anti-waves, we began laser physics experiments in 2020. We can expect that, if these experiments confirm infinite dimension orthogonality at the infinitesimal, we will be one step closer to the elucidation of field energy. This will be due to the fact that, as shown in Figure 34, black hole-level energy exists in infinitesimal space as a matrix vector, in the sense articulated by Feynman.[14]

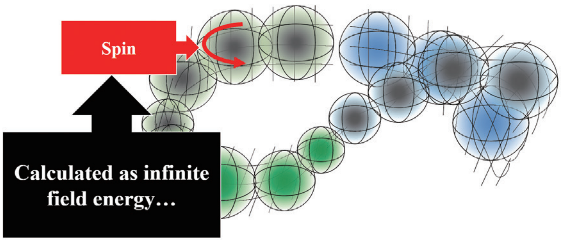

Figure 34: Infinite dimension matrix vectors crossing orthogonally at the infinitesimal

Acknowledgments

Our research has benefited tremendously from the astute guidance of Dr. Tadao Nakamura. We wish to express our deepest respect and gratitude to Dr. Nakamura for instantly perceiving our research's deep connection to quantum phenomena that exhibit Klein bottles and Möbius strips with gaps, and for teaching us that quantum gates acting as oracles can be expressed mathematically.

We are also deeply thankful to Softbank Group Corp. Representative Director Masayoshi Son, who entrusted us with the task of giving Pepper emotion and has supported our research tirelessly.

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